Large corrections to electroweak parameters in technicolor theories

B. Holdom and J. Terning
Department of Physics, University of Toronto, Toronto, Ontario, Canada M5T 2Z8
and Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA

Received 9 June 1990

We study the leading radiative corrections to various weak interaction parameters due to new heavy particles. We use an effective chiral lagrangian and input from low energy QCD to confirm the large size of these effects in technicolor theories.

Precision electroweak measurements are presently providing a sensitive probe for new physics at and above the scale of weak interactions. It has been stressed [1] that the effects of new heavy particles which do not couple directly to light fermions are manifest almost entirely in corrections to the vacuum polarization amplitudes of the electroweak gauge bosons. We find that an effective lagrangian analysis provides useful insight into the structure of these "oblique" corrections. And a related analysis for low energy QCD helps to determine the size of these corrections in technicolor theories.

We begin by considering any model of electroweak symmetry breaking in which there is some mass scale $\Lambda_\chi > M_Z$, below which the approximate $SU(2)_L\times SU(2)_R$ symmetry is realized nonlinearly, and above which are mass scales of new physics. This includes technicolor models where typically $\Lambda_\chi \approx 1$ TeV and standard scalar Higgs models as long as $m_H \gg M_Z$. Then for momenta $p<\Lambda_\chi$ the effective theory of the $SU(2)\times U(1)$ gauge bosons and their associated Goldstone bosons is described by a gauged chiral lagrangian.

The chiral lagrangian provides a systematic energy expansion in which derivatives and gauge fields both count as one power of momenta p. We will find that the leading corrections to weak interaction parameters occur in the $O(p^4)$ terms. And since we are interested only in vacuum polarizations we need only consider those terms of the chiral lagrangian which describe vertices with two gauge fields and no Goldstone bosons.

In the usual construction the Goldstone fields $\pi(x)\equiv\sum_i \pi_i(x)\tau_i$ ($\mathrm{Tr}(\tau_i\tau_j)=\frac{1}{2}\delta_{ij}$), appear in the field $U(x)\equiv\exp[2i\pi(x)/F]$ where $F\approx 250$ GeV. Under a global $SU(2)_L\times SU(2)_R$ transformation $U(x)\Rightarrow R^\dagger U(x)L$. If we ignore weak isospin breaking mass effects, then the most general set of terms of order p^2 and p^4 constrained by chiral symmetry and containing two gauge field vertices is

$$
\begin{aligned}
L_{\text{eff}} &= \tfrac{1}{4}F^2\,\mathrm{Tr}\,D_\mu U^\dagger D^\mu U + \tfrac{1}{4}W_{a\mu\nu}W_a^{\mu\nu} + \tfrac{1}{4}B_{\mu\nu}B^{\mu\nu} \\
&\quad + L_{10}\,gg' B_{\mu\nu}W_a^{\mu\nu}\,\mathrm{Tr}\,U^\dagger\tau_3 U\tau_a + L_{11}\,\mathrm{Tr}\,D^2U^\dagger D^2 U\,, \\
D_\mu U(x) &\equiv \partial_\mu U(x) - igW_{a\mu}(x)U(x)\tau_a \\
&\quad + ig'B_\mu(x)\tau_3 U(x)\,. \qquad (1)
\end{aligned}
$$

(Note that the $B-L$ part of the hypercharge generator Y commutes with $U(x)$.)

Thus in addition to the parameters of the standard model, g, g', and F, we have two new dimensionless parameters L_{10} and L_{11} (our naming convention will become clear below). We assume that all these quantities are renormalized at the Z mass. That is, all physics at higher energy scales has been integrated out and absorbed into the values of the parameters. Let us write the full vacuum polarization tensor between two gauge fields A and B with couplings g_A and g_B as

$$
i\Pi_{AB}^{\mu\nu}(k^2)\equiv ig_A g_B \Pi_{AB}(k^2)(g^{\mu\nu}-k^\mu k^\nu/k^2)\,, \qquad (2)
$$

and expand $\Pi_{AB}(k^2)=\Pi_{AB}^{(0)}+k^2\Pi_{AB}^{(1)}+\dots$.

The values of these quantities, renormalized at the Z mass, may be read off from L_{eff}. The first term in L_{eff} produces $\Pi_{W_1W_1}^{(0)}=\Pi_{W_3W_3}^{(0)}=\Pi_{BB}^{(0)}=-\Pi_{W_3B}^{(0)}=$

$\frac{1}{4}F^2$ via the diagrams in fig. 1 and this yields the masses $M_Z\cos\theta = M_W = \frac{1}{2}gF$. The L_{11} term gives no contribution to vacuum polarization since the sum of diagrams in fig. 2, each with a vertex from the L_{11} term, vanishes. Thus only L_{10} introduces a nontrivial vacuum polarization, and $\Pi^{(1)}_{W_3B} = L_{10}$.

The important point is that higher derivative terms in the chiral lagrangian are suppressed by powers of $(M_Z/\Lambda_\chi)^2$, and thus higher order terms in the expansion of the $\Pi_{AB}(k^2)$ are safely ignored. Thus all "oblique" corrections to the weak interactions due to isospin preserving new physics at mass scales of order Λ_χ and above must enter through the value of L_{10}.

Another way to see this is to consider the unrenormalized vacuum polarization amplitudes (but calculated with an infrared cutoff of order M_Z). We may identify the ultraviolet divergent terms in the momentum expansion of $\Pi^{u}_{AB}(k^2)$ by assuming that there is a renormalizable theory at some scale above Λ_χ. For example the infinite quantities $\Pi^{(1)u}_{W_1W_1} = \Pi^{(1)u}_{W_3W_3}$ and $\Pi^{(1)u}_{BB}$ are related to the coupling renormalizations for g and g' respectively. The quantities $\Pi^{(0)u}_{W_1W_1} = \Pi^{(0)u}_{W_3W_3} = \Pi^{(0)u}_{BB} = -\Pi^{(0)u}_{W_3B}$ may or may not be infinite depending on whether there is an elementary scalar in the theory above Λ_χ. If there is, then the infinity is related to the wave function renormalization of the scalar field. The quantity $\Pi^{(1)u}_{W_3B}$ on the other hand is finite since a two-derivative term which mixes W and B fields does not exist in the underlying renormalizable theory. Thus contributions to L_{10} are effectively cut off above the scale Λ_χ.

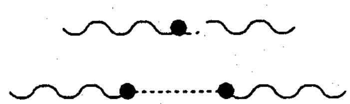

Fig. 1. Contributions to vacuum polarization. Round vertex is from the $O(p^2)$ term. The dashed line represents the Goldstone boson propagator.

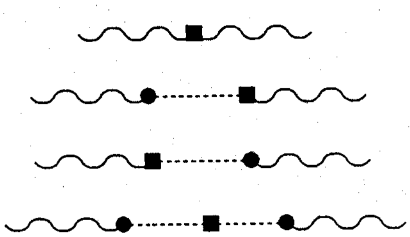

Fig. 2. Square vertex is from the L_{11} term.

In a recent review [1] Peskin provides formulas for physical quantities in terms of combinations of various unrenormalized vacuum polarization amplitudes (in a renormalizable theory with scalar fields). The various infinities in his expressions must of course cancel and this means that there can be no dependence on the various $\Pi^{(0,1)u}_{AB}$ except for $\Pi^{(1)u}_{W_3B}$. Since we are ignoring other finite contributions suppressed by $(M_Z/\Lambda_\chi)^2$ we see again that physical quantities can depend only on $\Pi^{(1)u}_{W_3B} = \Pi^{(1)}_{W_3B} = L_{10}$.

Thus far we have ignored weak isospin breaking mass effects, such as would occur with a very massive top quark or any other massive nondegenerate fermion doublet. Now the leading correction occurs in order p^2 terms in the effective lagrangian. The new terms produce a finite splitting between $(F^\pm)^2$ and $(F_3)^2$ proportional to $(m_t - m_b)^2$. This yields another physical quantity $\rho = 1 + \Delta\rho$ as measured by the relative strength of the charged and neutral weak currents near $q^2 = 0$ [1].

$$\Delta\rho \equiv \frac{e^2}{s^2c^2m_Z^2}\left(\Pi^{(0)}_{W_1W_1} - \Pi^{(0)}_{W_3W_3}\right) \approx \frac{3\alpha m_t^2}{16\pi s^2c^2m_Z^2} + \ldots . \tag{3}$$

The first term on the right-hand side is the contribution from a heavy top quark; the ... indicates all other contributions.

It is straightforward to simplify the formulas in ref. [1] by keeping only the k^0 and k^2 terms of the $\Pi_{AB}(k^2)$ and expressing the results in terms of L_{10} and $\Delta\rho$.

$$s_*^2(m_Z^2) - s^2 = -\frac{e^2}{c^2-s^2}L_{10} - \frac{s^2c^2}{c^2-s^2}\Delta\rho , \tag{4}$$

$$\sin^2\theta_w|_S - s^2 = -(m_W^2/m_Z^2 - c^2)$$
$$= -\frac{2e^2c^2}{c^2-s^2}L_{10} - \frac{c^4}{c^2-s^2}\Delta\rho , \tag{5}$$

$$Z_* = 1 - \frac{e^2}{s^2c^2}L_{10} , \tag{6}$$

$$c \equiv \cos\theta_w|_Z , \quad s \equiv \sin\theta_w|_Z , \tag{7}$$

$\theta_w|_Z$ is defined in terms of physical quantities according to the "Z-standard" [1]

$$\sin 2\theta_w|_Z \equiv [4\pi\alpha_{*,0}(m_Z^2)/\sqrt{2}G_F m_Z^2]^{1/2} \tag{8}$$

$\alpha^{-1}_{*,0}(m_Z^2) \approx 129$ is the value of the running electromagnetic coupling including renormalization effects due only to observed quarks and leptons. Then $\sin^2\theta_w|_Z \approx 0.232$. The Sirlin definition is

$$\sin^2\theta_w|_S \equiv 1 - m_W^2/m_Z^2 . \tag{8'}$$

$s_*^2(m_Z^2)$ determines the polarization asymmetry at the Z pole

$$A_{LR} = \frac{\sigma(e_L^- e^+ \Rightarrow Z) - \sigma(e_R^- e^+ \Rightarrow Z)}{\sigma(e_L^- e^+ \Rightarrow Z) + \sigma(e_R^- e^+ \Rightarrow Z)} \approx \frac{2 - 8s_*^2(m_Z^2)}{1 + [1 - 4s_*^2(m_Z^2)]^2} . \tag{9}$$

The forward–backward asymmetries A_{FB}^f depend on the final state flavor and are also determined by $s_*^2(m_Z^2)$ [1]. Z_* is a factor which renormalizes the Z propagator and multiplies the Z width [1].

A useful check on the result in (4) is provided by the quark loop calculations in chiral quark models [2]. In these models the coefficients of various $O(p^4)$ terms in a low energy effective theory are obtained and in particular

$$L_{10}^{CQM} = -N_c/96\pi^2 . \tag{10}$$

This with (4) and $\Delta\rho = 0$ yields the same result for $s_*^2(m_Z^2) - \sin^2\theta_w|_Z$ as given in ref. [1] for a degenerate massive quark doublet.

We would like to estimate $L_{10}(M_Z)$ in technicolor theories. (We now make explicit the dependence of L_{10} on the renormalization scale.) We write

$$L_{10}(M_Z) \equiv \Delta L_{10} + L_{10}(\Lambda_\chi) , \tag{11}$$

since it is possible to extract ΔL_{10} from the literature. The vacuum polarizations due to technipions with masses $m_{TP} < \Lambda_\chi$ have been calculated [3] in a typical technicolor theory and these results may be translated into a value for ΔL_{10}.

$L_{10}(\Lambda_\chi)$ on the other hand must match onto the underlying technicolor theory at the scale Λ_χ. In principle it is obtained by integrating out technihadrons with masses of order Λ_χ and heavier. It is usually argued that ΔL_{10} will dominate $L_{10}(\Lambda_\chi)$ due to a $\ln(\Lambda_\chi/m_{TP})$ factor in the former. Such arguments have to be treated with some caution; for example the opposite could be argued in the large-N_{tc} limit (where N_{tc} is the number of technicolors) since $L_{10}(\Lambda_\chi)$ and ΔL_{10} are of $O(N_{tc})$ and $O(1)$ respectively [4]. We will argue that low energy QCD provides a useful estimate of $L_{10}(\Lambda_\chi)$.

We therefore look more closely at an example model with one family of techniquarks and technileptons having approximate $SU(8)\times SU(8)$ global symmetry. Besides the three Goldstone bosons there are 4 color singlet, 32 color octet, and 24 color triplet technipions of masses m_1, m_8, and m_3 respectively. we translate the results of ref. [3] into the following:

$$\Delta L_{10} \approx -(2f_1 + 8f_8 + 6f_3) ,$$

$$f_i = \frac{1}{96\pi^2} \ln(\Lambda_\chi/\mathrm{Max}(m_i, M_Z)) . \tag{12}$$

Basically each charged pair contributes one unit of the appropriate f_i. We have included the Goldstone bosons; they contribute one unit of f_1 via diagrams without internal gauge boson lines.

There is model dependence both in the numbers of technipions and in the masses m_i. All of the colored and perhaps also the charged color-singlet technipions receive most of their mass from first order $SU(3)\times SU(2)\times U(1)$ corrections. In a walking technicolor theory all these mass contributions will be substantially increased [5] (as well as mass contributions arising from four technifermion operators), thereby decreasing $|\Delta L_{10}|$. But for illustration we give two estimates, the first for $m_1 < M_Z$, $m_8 = 245$ GeV, $m_3 = 160$ GeV as given in ref. [6] and the second for m_8 and m_3 three times as large. And we take $\Lambda_\chi \approx 1$ TeV which is twice a typical techifermion mass, Then

$$\Delta L_{10} = -0.029, \ -0.012. \tag{13}$$

We now estimate $L_{10}(\Lambda_\chi)$. Here we note that the L_{10} term in our effective theory has a counterpart of exactly the same form in the chiral lagrangian describing low energy QCD (and this is the origin of our notation). L_{10}^{QCD} has been experimentally determined by the analysis of Gasser and Leutwyler from the pion charge radius and the decay $\pi \Rightarrow e\nu\gamma$ [4]. We take $\Lambda_\chi^{QCD} \approx 660$ MeV (twice the constituent quark mass), and $L_{10}^{QCD}(\Lambda_\chi^{QCD}) \approx -5.4 \pm 0.3 \times 10^{-3}$ [4,7].

To carry this result over to technicolor we just need some idea of the dependence on the number of technifermion doublets and technicolors. We note that $L_{10}^{QCD}(\Lambda_\chi^{QCD})$ is about 1.75 times the value quoted in (10) for L_{10}^{CQM}. The chiral quark models actually do a fair job of obtaining five of the ten L_i in the Gasser–Leutwyler lagrangian. But of the five L_i predicted, $L_{10}^{\rm CQM}$ is the worst. Recently the chiral quark model has been improved by the introduction of a more realistic momentum dependent dynamical quark mass [8]. In this model all ten L_i may be obtained and they all (including L_{10}) are remarkably consistent with the experimental values.

In view of the success of these models, we will carry over the implied dependence on number of doublets ($N_{\rm d}$) and technicolors. We therefore estimate

$$L_{10}(\Lambda_\chi)\approx N_{\rm d}\cdot\tfrac{1}{3}N_{\rm tc}L_{10}^{\rm QCD}(\Lambda_\chi^{\rm QCD})\,. \qquad (14)$$

For example in the model described $N_{\rm d}=4$ and if we take $N_{\rm tc}=4$ then $L_{10}(\Lambda_\chi)\approx-0.029$. Perhaps a more realistic model [9] has two technifamilies with $N_{\rm tc}=2$; but this gives the same result. The same result also applies if different technifermion doublets have somewhat different masses, as long as all the masses are well above $M_{\rm Z}$. On the other hand $|L_{10}(\Lambda_\chi)|$ may be somewhat less in a walking theory, as seen for example from $L_{10}^{\rm CQM}$ which corresponds to a momentum independent quark mass. We therefore consider the following range typical:

$$L_{10}(\Lambda_\chi)\approx-0.02 \text{ to } -0.03\,. \qquad (15)$$

From this we conclude that $L_{10}(\Lambda_\chi)$ and ΔL_{10} have the same sign and that they are likely to be of the same order of magnitude. Combining the estimates we arrive at the following typical shifts in the quantities $A_{\rm LR}$, $M_{\rm W}$, and Z_* due to $L_{10}(M_{\rm Z})$:

$$\Delta A_{\rm LR}|_{L_{10}}\approx-0.065(L_{10}/-0.045)\,,$$

$$\Delta M_{\rm W}|_{L_{10}}\approx-660(L_{10}/-0.045)\ {\rm MeV}\,,$$

$$\Delta Z_*|_{L_{10}}\approx0.025(L_{10}/-0.045)\,. \qquad (16)$$

The shifts in these quantities due to $\Delta\rho$ are

$$\Delta A_{\rm LR}|_{\Delta\rho}\approx0.026(\Delta\rho/0.01)\,,$$

$$\Delta M_{\rm W}|_{\Delta\rho}\approx570(\Delta\rho/0.01)\ {\rm MeV}\,,$$

$$\Delta Z_*|_{\Delta\rho}\approx0\,. \qquad (17)$$

Thus the shifts due to L_{10} are substantial and may easily dominate the $\Delta\rho$ contribution to $\Delta A_{\rm LR}$. The same is true for shifts in forward–backward asymmetries at the Z pole, where for charge $-\frac{1}{3}$ quarks, charge $\frac{2}{3}$ quarks, and charged leptons the shifts are respectively about 0.71, 0.55, and 0.21 as large as $\Delta A_{\rm LR}$. The L_{10} and $\Delta\rho$ contributions in $\Delta M_{\rm W}$ may more or less cancel, but any measurement of ΔZ_* could help to disentangle the two effects. Perhaps most important is the fact that complete cancellation between L_{10} and $\Delta\rho$ contributions cannot occur simultaneously in both $\Delta A_{\rm LR}$ and $\Delta M_{\rm W}$.

In summary we have associated an important class of "oblique" radiative corrections to weak interaction parameters with one $O(p^4)$ term in an effective chiral lagrangian. These corrections are large in technicolor theories with one or more technifamilies since technipions and the remaining technihadrons give comparable contributions of the same sign. Of course the bulk of these corrections would be avoided in a technicolor model with only one color singlet technifermion doublet. Then $L_{10}(M_{\rm Z})\approx L_{10}(\Lambda_\chi)\approx-0.005$ and the various shifts are as much as ten times smaller. Nevertheless we have confirmed that a wide class of technicolor models with a rich particle content have a distinctive signature for precision electroweak measurements.

B.H. thanks A. Cohen for a discussion. This research was supported in part by the Natural Sciences and Engineering Research Council of Canada and by the National Science Foundation under Grant No. PHY89-04035, supplemented by funds from the National Aeronautics and Space Administration.

Note added. We have since received the preprints [10] in which closely related work is described.

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